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The network operator method for search of the most suitable mathematical expression

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Introduction

The network operator method is developed to search for the most suitable mathematical expression in the problems of approximation, identification, synthesis of control system structure etc.

Most of the problems of search of mathematical expressions are solved using manual-automatic mode. The researcher manually writes down the mathematical expression with symbolic parameters. Then the computer performs the search of optimal values of parameters.

The network operator method helps to automate the search of mathematical expression. The researcher defines the sets of parameters, variables and operations of mathematical expression, and the computer generates the set of mathematical expressions that satisfy given restrictions. Then using the optimization algorithm the computer finds suitable mathematical expression and its parameters.

Here the problem of effective presentation of mathematical expression in the computer memory occurs. Nowadays the symbolic notation is widely used. The symbolic notation requires strictly defined symbols of operations, parameters, variables and their order. To calculate the result of symbolic notation the special analytical software is needed. It decodes mathematical expression and performs the calculation.

In this paper the new form of notation of mathematical expression is given. The mathematical expression is presented as a directed graph which is called the network operator. The network operator contains all necessary information to calculate the result of mathematical expression: operations, parameters and variables as well as the order of calculation.

In the computer memory the network operator can be presented in different forms. In the given paper the two possible forms are considered. These forms are special integer matrices and vectors that are based on incident matrices and vectors respectively. Both forms do not need any analytical software which helps to make calculations more effective in comparison to symbolic notations.

This paper contains full description of the network operator method. The properties of the network operator investigated and examples of application are given. While solving the problems by applying the network operator method the authors wrote many programs for PC and got state certificates. Possible applications of the network operator method are various and are currently investigated. For example the network operator can be used for the search of the approximant solution of any variation problems, inverse problems, integrals, integral and differential equations etc.

Definition 0.1. Mathematical expression is a notation of mathematical symbols that has only way for calculations

The search of mathematical expression by a computer can be performed using genetic programming (GP). Genetic programming [1] uses symbolic notation to present possible solutions. Each symbol in the notation is either operation or numerical parameter. The main advantage of genetic programming in comparison to genetic algorithm is crossover operation for the strings of different length. Though GP has certain disadvantages: the necessity of substring search to perform crossover operation, the growth of length of notation, and the complexity of search of optimal parameters.

1. Program notations of mathematical expressions

Consider the structure of mathematical expression. Mathematical expressions consist of unary and binary operations, parameters and variables. To construct mathematical expressions we use four constructive sets.

Set of variables

$$\boldsymbol{X} = (x_1, \ldots, x_N),\ x_i \in \boldsymbol{R}^1,\ i = \overline{1, N}. \tag{1.1}$$

Set of parameters

$$\boldsymbol{Q} = (q_1, \ldots, q_P),\ q_i \in \boldsymbol{R}^1,\ i = \overline{1, P}. \tag{1.2}$$

Set of unary operations

$$\boldsymbol{O}_1 = (\rho_1(z) = z, \rho_2(z), \ldots, \rho_W(z)). \tag{1.3}$$

Set of binary operations

$$\boldsymbol{O}_2 = (\chi_0(z', z''), \ldots, \chi_{V-1}(z', z'')). \tag{1.4}$$

It is necessary to have identical operation among unary operations.

$$\rho_1(z) = z. \tag{1.5}$$

Binary operations should be commutative

$$\chi_i(z', z'') = \chi_i(z'', z'),\ i = \overline{0, V-1}, \tag{1.6}$$

associative

$$\chi_i(z', \chi_i(z'', z''')) = \chi_i(\chi_i(z', z''), z'''),\ i = \overline{0, V-1}, \tag{1.7}$$

and have a unit element

$$\forall \chi_i(z', z'') \in \boldsymbol{O}_2\ \exists e_i\ \Rightarrow\ \chi_i(e_i, z) = z,\ i = \overline{0, V-1}. \tag{1.8}$$

Definition 1.1. Program notation of mathematical expression is a notation of expression with the elements of constructive sets (1.1) – (1.4).

2. Graphical notations of mathematical expressions.

To present mathematical expression as a graph we use the program notation of this expression. Let us add to program notation unary and binary operations that do not influence the calculations, but set given the order of operations in the notation. Identity unary operation $\rho_1(z) = z$ and binary operation with a unit element $\chi_i(e_i, z) = z$ set such an order in the program notation that only unary operations can be arguments to binary operations, and binary operations, parameters or variables can be arguments to unary operations.

Definition 2.1. Graphic notation of mathematical expression is the notation of binary operation that fulfills the following conditions:

a) binary operation can have unary operations or unit element of this binary operation as its arguments;

b) unary operation can have binary operation, parameter or variable as its argument;

c) binary operation cannot have unary operations with equal constants or variables as its arguments.

Theorem 2.1. Any program notation of mathematical expression can be presented as a graphic notation.

3. Network operators of mathematical expressions.

We use graphic notation to construct the graph of mathematical expression.

While constructing the graph we follow the rule: unary operations correspond to the edges, and binary operations, parameters or variables correspond to the nodes of the graph. The detailed description of construction of elements of the graph is given at Fig. 3.1

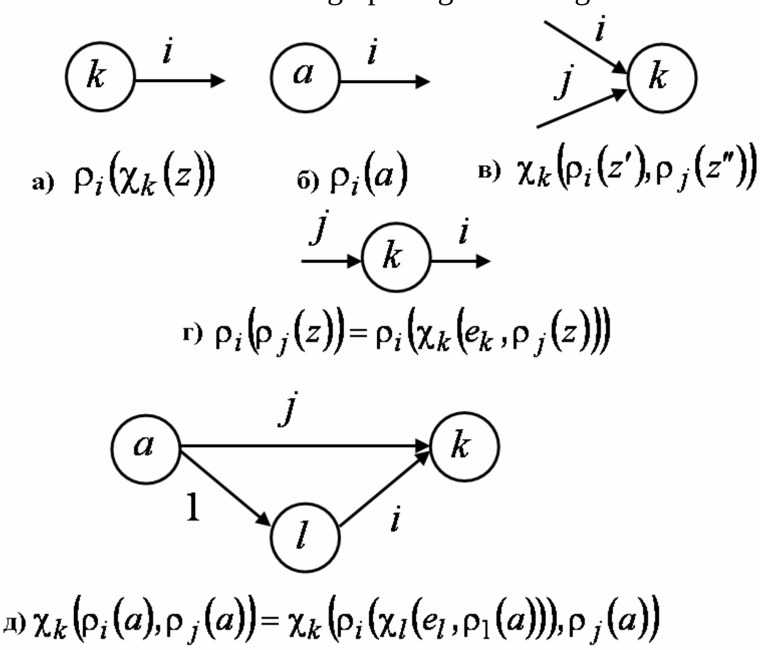

Fig. 3.1. The construction of elements of the graph

Definition 3.1 Network operator is the directed graph that has the following properties:

a) graph has no loops;

b)any nonsource node has at least one edge from the source node;

c) any non sink node has at least one edge to sink node;

d) every source node corresponds to the element of set of variables $\mathbf{X}$ or of set of parameters $\mathbf{Q}$;

e) every node corresponds to binary operation of set of binary operations $\mathbf{O}_2$;

f) every edge corresponds to unary operation of set of unary operations $\mathbf{O}_1$.

Theorem 3.1. Suppose we construct the network operator using its graphical notation. Then to calculate the mathematical expression it is enough to fulfill the rules:

a) to calculate unary operation only for the edge that comes out of the node that has no incoming edges;

b) to delete the edge from the graph if its unary operation has already been calculated;

c) to calculate binary operation right after unary operation that corresponds to the edge incoming this node,

d) to terminate calculations when all edges are deleted from the graph.

Consider an example of construction of the network operator for mathematical expression

$$y=x_1+\sin(x_1)+q_1x_1e^{-x_2}.$$

Let us set the following unary and binary operations $\rho_1(z)=z$, $\rho_3(z)=-z$, $\rho_6(z)=e^z$, $\rho_{12}(z)=\sin z$, $\chi_0(z',z'')=z'+z''$, $\chi_1(z',z'')=z'z''$. Then we obtain the following program notation of mathematical expression

$$y=\chi_0(\chi_0(x_1,\rho_{12}(x_1)),\chi_1(\chi_1(q_1,x_1),\rho_6(\rho_3(x_2)))).$$

If we add operations $\rho_1(z)$ and $\chi_0(z', z'')$ with a unit element 0, we shall get the graphic notation of mathematical expression:

$$y = \chi_0(\rho_1(\chi_0(\rho_1(\chi_0(\rho_1(x_1),0)),\rho_{12}(x_1))),$$
$$\rho_1(\chi_1(\rho_1(\chi_1(\rho_1(q_1),\rho_1(x_1))),\rho_6(\chi_0(\rho_3(x_2),0))))).$$

See Fig. 3.2 for the graph of network operator for the mathematical expression above.

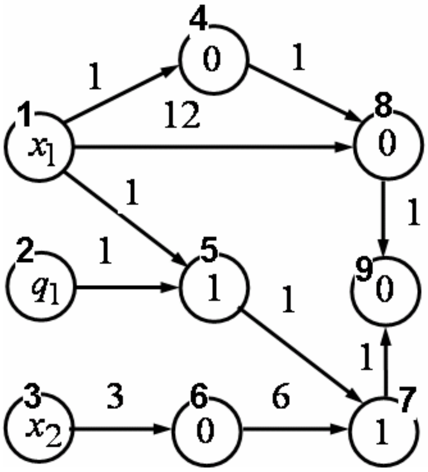

Fig. 3.2. Graph of network operator

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