MULTI-OBJECTIVE STRUCTURAL PARAMETRICAL SYNTHESIS OF OPTIMAL CONTROL SYSTEM BY NETWORK OPERATOR METHOD

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Abstract— **The problem of multi-objective synthesis of optimal control is considered. It is necessary to find the control as a function of problem space coordinates. Initial values of the control object are in closed bounded domain. Functionals of control quality include multiple integrals over domain initial values. To solve the problem of multi-objective synthesis we use numerical method that is based on genetic algorithm and network operator. This method allows automatic search of mathematical equations that describe dependence of control from the problem space coordinates. The solution of the problem of synthesis of space vehicle descent with indeterminate initial conditions of entry in the dense layers of atmosphere is given.**

I. INTRODUCTION

When synthesizing a control system, generally analytical approach to mathematical model of object and functional is often used. As a result of analysis of the problem the researcher defines the behavior of optimal system and forms the appropriate control according to it.

Analytical approaches to synthesis have some difficulties when applied to complex nonlinear mathematical models of object and functionals.

Until recently numerical methods of synthesis of control systems have been based on mathematical programming where the structure of control system was given in advance and the optimal parameters were to be found. This approach is widely used in construction of stabilization system near optimal program trajectory that was obtained as a solution of optimal control problem.

When creating stabilization system, the mathematical model of the object is often linearized on optimal trajectory. That means that in most cases the structure of control system includes linear regulators which parameters are found in the process of synthesis. This approach cannot be called the method of synthesis since the structure of control system is limited by blocks of certain types and their connection is not formalized.

The recent computational technologies have developed dramatically and opportunity to create a group of numerical methods of control synthesis has occurred. These methods of control synthesis are aimed to create the optimal structure of control systems and search optimal values of its parameters. The methods use special data structure that allows to describe universal mathematical equations, store and operate them into PC. Genetic programming [1-3] and the network operator method [4] can be included in that group.

This methods differ in data structures that they use to store mathematical equations. Genetic programming uses Polish notation, network operator method applies integer matrix. Matrix does not require any lexical analysis and works more quickly with calculations of equations.

Numerical methods of synthesis despite their effectiveness have certain disadvantages. For example they lack analytical research of the optimal control system behavior. The experiments carried out such synthesized control systems revealed some common features:

a) the result of numerical synthesis is a set of different control systems that are close to optimum and have almost identical parameters of quality.

b) synthesized systems depend on problem space coordinates, but sometimes they are sensitive to variations of the model.

To improve the quality of synthesis we should include condition of less sensitivity to variations in problem statement. For numerical method this complication of problem statement will not lead to significant time consumption. In the given article we present further development of numerical method of network operator for multi-objective synthesis of control system when we take into consideration variations of initial values of the system.

II. PROBLEM STATEMENT

The following problem of synthesis of control is considered.

The system of differential equations which describes the dynamics of the object is given

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u}), \tag{2.1}$$

where $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{u} \in \boldsymbol{U} \subseteq \mathbb{R}^m$, $\boldsymbol{U}$ - closed bounded domain.

The domain of existence of initial values is given

$$\mathbf{x}(0) = \mathbf{x}^0,\ \mathbf{x}^0 \in \boldsymbol{X}_0 \subseteq \mathbb{R}^n, \tag{2.2}$$

where $\mathbf{x}^0 = \left[x_1^0 \dots x_n^0\right]^T$, $\mathbf{x}^0 \in \boldsymbol{X}_0$.

It is necessary to find the control in the following form

$$\mathbf{u} = \mathbf{g}(\mathbf{x}), \tag{2.3}$$

to satisfy boundary conditions

$$\mathbf{u} \in \boldsymbol{U}, \tag{2.4}$$

and minimize the functionals

$$J_i = \int_{\boldsymbol{X}_0} \dots \int \int_0^{t_f} f_{0,i}(\mathbf{x},\mathbf{u}) dt dx_1^0 \dots dx_n^0 \to \min, \tag{2.5}$$

$i = \overline{1,N}$,

where t_f is the given time.

Solution of this problem is the set of Pareto

$$\tilde{\boldsymbol{P}} = \left\{\tilde{\mathbf{g}}^i(\mathbf{x}) : i = \overline{1,\tilde{K}}\right\}. \tag{2.6}$$

For any possible solution $\mathbf{u} = \mathbf{g}(\mathbf{x})$ $\exists \tilde{\mathbf{g}}^i(\mathbf{x}) \in \tilde{\boldsymbol{P}}$, so that $\mathbf{J}(\tilde{\mathbf{g}}^i(\mathbf{x})) \le \mathbf{J}(\mathbf{g}(\mathbf{x}))$ or $\mathbf{J}(\tilde{\mathbf{g}}^i(\mathbf{x})) = \mathbf{J}(\mathbf{g}(\mathbf{x}))$, where $\mathbf{J}(\mathbf{g}(\mathbf{x}))$ is a vector of values of the functionals (2.5) for $\mathbf{g}(\mathbf{x}), \mathbf{J}(\mathbf{g}(\mathbf{x})) = \left[J_1(\mathbf{g}(\mathbf{x})) \dots J_N(\mathbf{g}(\mathbf{x}))\right]^T$.

If $\forall \mathbf{g}'(\mathbf{x}), \mathbf{g}''(\mathbf{x})$ inequality is fulfilled

$$\mathbf{J}(\mathbf{g}'(\mathbf{x})) \le \mathbf{J}(\mathbf{g}''(\mathbf{x})),$$

then $J_i(\mathbf{g}'(\mathbf{x})) \le J_i(\mathbf{g}''(\mathbf{x}))$, $i = \overline{1,N}$, and $\exists k$, $1 \le k \le N$ $J_k(\mathbf{g}'(\mathbf{x})) < J_k(\mathbf{g}''(\mathbf{x}))$.

The problem (2.1) – (2.6) let us call the main problem of multi-objective synthesis of optimal control system.

If in the problem (2.1) - (2.6) we use initial values for one vector

$$\boldsymbol{X}_0 = \left\{\mathbf{x}^0\right\}, \tag{2.7}$$

then it is the classical problem of multi-objective synthesis of optimal control system.

In the classical problem we use ordinary functionals

$$\bar{J}_i = \int_0^{t_f} f_{0,i}(\mathbf{x},\mathbf{u}) dt \to \min, \tag{2.8}$$

Solution of the classical problem of multi-objective synthesis is Pareto set too

$$\bar{\boldsymbol{P}} = \left\{\bar{\mathbf{g}}^i(\mathbf{x}) : i = \overline{1,\bar{K}}\right\}$$

Theorem 1. Assume $\bar{\boldsymbol{P}}$ and $\tilde{\boldsymbol{P}}$ are the sets of Pareto have been obtained for the classical and main problems sequence of multi-objective synthesis, and it is fulfilled the condition $\bar{\mathbf{x}}^0 \in \boldsymbol{X}_0$, where $\bar{\mathbf{x}}^0$ is initial values in the classical problem.

Then $\tilde{\boldsymbol{P}} \supset \bar{\boldsymbol{P}}$.

Proof. Suppose the proposition of the theorem is not fulfilled, $\exists \tilde{\mathbf{g}}(\mathbf{x}) \in \tilde{\boldsymbol{P}}$ and $\tilde{\mathbf{g}}(\mathbf{x}) \notin \overline{\boldsymbol{P}}$. Then $\exists \overline{\mathbf{g}} \in \overline{\boldsymbol{P}}$ for which $\overline{\mathbf{J}}(\overline{\mathbf{g}}(\mathbf{x})) \le \overline{\mathbf{J}}(\tilde{\mathbf{g}}(\mathbf{x}))$. We note that then $\tilde{\mathbf{J}}(\overline{\mathbf{g}}(\mathbf{x})) \le \tilde{\mathbf{J}}(\tilde{\mathbf{g}}(\mathbf{x}))$ too.

Let us create new solution of main problem of synthesis

$$\hat{\mathbf{g}}(\mathbf{x}) = \left(1 - \varsigma\left(\left\|\mathbf{x}(0) - \overline{\mathbf{x}}^0\right\|\right)\right)\tilde{\mathbf{g}}(\mathbf{x}) + \\ + \varsigma\left(\left\|\mathbf{x}(0) - \overline{\mathbf{x}}^0\right\|\right)\overline{\mathbf{g}}(\mathbf{x}),$$

where $\varsigma(a)$ is an indicator function $\varsigma(a) = \begin{cases} 0, \text{ if } a \neq 0 \\ 1, \text{ otherwise} \end{cases}$,

Then $\hat{\mathbf{g}}(\mathbf{x}) = \overline{\mathbf{g}}(\mathbf{x})$, if in moment $t = 0$ $\mathbf{x}(0) = \overline{\mathbf{x}}^0$.

But then the following condition is right

$$\tilde{\mathbf{J}}(\hat{\mathbf{g}}(\mathbf{x})) \le \tilde{\mathbf{J}}(\tilde{\mathbf{g}}(\mathbf{x})),$$

thus $\tilde{\mathbf{g}}(\mathbf{x}) \notin \tilde{\boldsymbol{P}}$. So $\tilde{\boldsymbol{P}}$ is not the solution of the main problem of synthesis. ■

From Theorem 1 it follows that when solving the main problem of multi-objective synthesis (2.1) – (2.6) we obtain mathematical equation that describes the functional dependence of control from problem space coordinates $\mathbf{u} = \tilde{\mathbf{g}}(\mathbf{x})$. This control has no worse values of quality functional (2.8) than any solution of classical problem of synthesis for particular initial values $\mathbf{x}(0) = \tilde{\mathbf{x}}^0 \in \boldsymbol{X}_0$.

For numerical solution of the problem of multi-objective synthesis it is necessary to create the space of different functions including piecewise continuous and not continuously differentiable ones. From this space we take functions that belong to Pareto set. Such space of functions can be created with the help of network operator.

III. NETWORK OPERATOR

Mathematical equation consists of variables, parameters, unary and binary operations that form four constructive sets.

Set of variables

$$\boldsymbol{X} = (x_1, \ldots, x_n),\ x_i \in \mathbb{R}^1. \tag{3.1}$$

Set of parameters

$$\boldsymbol{Q} = (q_1, \ldots, q_P),\ q_i \in \mathbb{R}^1,\ i = \overline{1, P}. \tag{3.2}$$

Unary operations set

$$\boldsymbol{O}_1 = (\rho_1(z) = z, \rho_2(z), \ldots, \rho_W(z)). \tag{3.3}$$

Binary operations set

$$\boldsymbol{O}_2 = (\chi_0(z', z''), \ldots, \chi_{V-1}(z', z'')). \tag{3.4}$$

Unary operations set must have an identity operation $\rho_1(z) = z$. Binary operations must be commutative $\chi_i(z', z'') = \chi_i(z'', z')$, $i = \overline{0, V-1}$, associative $\chi_i(z', \chi_i(z'', z''')) = \chi_i(\chi_i(z', z''), z''')$, $i = \overline{0, V-1}$, and have unit element $\forall \chi_i(z', z'') \in \boldsymbol{O}_2\ \exists e_i \Rightarrow \chi_i(e_i, z) = z$, $i = \overline{0, V-1}$.

Network operator is a directed graph with following properties:

0) graph should be circuit-free;

1) there should be at least one edge from the source node to any non-source node;

2) there should be at least one edge from any non-source node to sink node;

3) every source node corresponds to the item of set of variables $\boldsymbol{X}$ or parameters $\boldsymbol{Q}$;

4) every non-source node corresponds to the item of binary operations set $\boldsymbol{O}_2$;

5) every edge corresponds to the item of unary operations set $\boldsymbol{O}_1$.

To present the network operator in the memory of PC we use an integer matrix. Let us numerate all nodes of network operator so that the number of node from which the edge comes out is smaller than the number of the node this edge comes in. Such numeration can always be done for oriented circuit-free graph. If we replace diagonal elements of the vertex incident matrix with numbers of binary operations that correspond to appropriate nodes and nonzero nondiagonal elements with numbers of unary operations, we shall get network operator matrix $\boldsymbol{\Psi} = [\psi_{ij}]$, $i, j = \overline{1, L}$.

Network operator matrix (NOM) is an integer upper-triangular matrix that has as its diagonal elements numbers of binary operations and off-diagonal elements are zeros or numbers of unary operations, besides if we replace diagonal elements with zeros and nonzero off-diagonal elements with ones we shall get an incident matrix of the graph that satisfies conditions 0)-2) of network operator definition.

To calculate the mathematical equation with the help of NOM let us introduce three integer vectors:

- vector of numbers of nodes for variables $\mathbf{b} = [b_1 \ldots b_N]^T$, where b_i is the number of source node in the network operator that is linked to variable x_i;

- vector of numbers of nodes for parameters $\mathbf{s} = [s_1 \ldots s_P]^T$, where s_i is the number of the source node in the network operator that is linked to parameter q_i;

- vector of numbers for output variables $\mathbf{d} = [d_1 \ldots d_M]^T$, where d_i is the number of the node in the network operator that is linked to output variable y_i, M is the quantity of output variables.

Theorem 2 If we have the NOM $\mathbf{\Psi} = [\psi_{ij}]$, $i, j = \overline{1, L}$, and vectors of numbers of nodes for variables $\mathbf{b} = [b_1 \ldots b_n]^T$, parameters $\mathbf{s} = [s_1 \ldots s_P]^T$ and outputs $\mathbf{d} = [d_1 \ldots d_m]^T$ then we can calculate the mathematical expression described by NOM.

Proof. To store intermediate results let us introduce vector of nodes $\mathbf{z} = [z_1 \ldots z_L]^T$. Set initial values to vector of nodes.

$$z_i^{(0)} = \begin{cases} x_k, \text{ if } i = b_k, \ k = \overline{1, n} \\ c_j, \text{ if } i = s_j, \ j = \overline{1, P} \\ e_{\psi_{ii}}, \text{ if } i \notin \mathbf{I}_0, \ \chi_{\psi_{ii}}(e_{\psi_{ii}}, z) = z \end{cases}, \qquad (3.5)$$

$i = \overline{1, L}$,

where $\mathbf{I}_0 = \{s_i, b_j : i = \overline{1, n}, j = \overline{1, P}\}$ - set of source nodes.

Calculate according to the following:

$$z_j^{(i)} = \begin{cases} \chi_{\psi_{jj}}\left(z_j^{(i-1)}, \rho_{\psi_{ij}}\left(z_i^{(i-1)}\right)\right), \text{ if } \psi_{ij} \neq 0 \\ z_j^{(i-1)}, \text{ otherwise} \end{cases}, \qquad (3.6)$$

where $i = \overline{1, L-1}$, $j = \overline{i+1, L}$.

For nonzero element ψ_{ij} of NOM $\mathbf{\Psi}$ we perform unary operation that corresponds to the edge (i, j), and binary operation that corresponds to node j, therefore all operations of network operator will be performed but for the first node that can be only source node. Source node can be a variable or parameter.

According to numeration of nodes $k > j > i$ unary operations for edges (j, k), that comes out from the node j can be performed after unary operations are performed for edges (i, j), that comes in the node j.

Equation (3.6) performs all operations in network operator and keeps the order of calculations, therefore, calculates the mathematical expression described by NOM. ■

To solve the problem of search of mathematical equation for control system synthesis let us construct the set of network operators

$$\Xi = \{\mathbf{\Psi}^i, i = \overline{1, H}\}. \qquad (3.7)$$

Then with the help of computational algorithm we search for optimal network operator $\tilde{\mathbf{\Psi}} \in \Xi$ that satisfies the conditions of the problem and values of parameters $\tilde{\mathbf{q}} = [\tilde{q}_1 \ldots \tilde{q}_P]^T$ for it.

The construction of (3.7) requires high-volume computational systems for integer matrix storage. To construct (3.7) we use method of basic solution variation.

IV. METHOD OF BASIC SOLUTION VARIATION

For network operator the following variations are defined:

0) replacement of unary operation on the edge;
1) replacement of binary operation in the node;
2) addition of an edge with a unary operation;
3) addition of a node with a binary operation;
4) deletion of the edge;
5) deletion of the node.

Variations (0)-(5) do not change the properties of network operator and thus do not influence on its correctness. If we perform variations (4)-(5) the properties should be taken into consideration. The deletion of an edge can occur only if there is at least one more edge that has the same source node and the sink node for deleted edge has at least one more incoming edge. Addition of a node requires the addition of at least two edges that in and outcome that node. Incoming edge should outcome from the node that is before the new node and the output edge should come into the next node on the way. Deletion of the node should come together with deletion of in and output edges.

All variations on the network operator can be presented as an integer variation vector that consists of four elements:

$$\mathbf{w} = [w_1 \;\; w_2 \;\; w_3 \;\; w_4]^T, \qquad (4.1)$$

where w_1 is the number of variation, w_2 is the number of row in NOM, w_3 is the number of column in NOM, w_4 is the number of unary or binary operation. Element w_4 depends on element w_1.

To construct a set of mathematical equations we define one basis mathematical equation that is described by basic network operator and set of variation vectors

$$\Xi_l = \left\{\mathbf{\Psi}^0, \boldsymbol{W}^i = \left(\mathbf{w}^{1,i}, \ldots, \mathbf{w}^{l,i}\right), i = \overline{1,H}\right\}, \qquad (4.2)$$

where $\mathbf{\Psi}^0$ is the matrix of basic network operator, $\boldsymbol{W}^i = \left(\mathbf{w}^{1,i}, \ldots, \mathbf{w}^{l,i}\right)$ is the set of variation vectors, l is the given length of set of variation vector.

Network operator $\mathbf{\Psi}^i$ from set (4.2) is obtained as a result of variation of basic solution.

$$\mathbf{\Psi}^i = \mathbf{w}^{l,i} \circ \ldots \circ \mathbf{w}^{1,i} \circ \mathbf{\Psi}^0. \qquad (4.3)$$

Application of set of variation vectors not only reduces the volume of required memory, but also allows to use genetic algorithm for the search of optimal solution

V. GENETIC ALGORITHM

To construct genetic algorithm we need to create basic solution described by NOM $\mathbf{\Psi}^0 = \left[\psi_{ij}^0\right]$, $i, j = \overline{1,L}$. Chromosome is an ordered set of variation vectors $\boldsymbol{W}^i = \left(\mathbf{w}^{1,i}, \ldots, \mathbf{w}^{l,i}\right)$, $i = \overline{1,H}$, where H is the dimension of population, l is the length of chromosome.

To improve the search we replace basic solution by the best found one after some generations that we call epoch.

Genetic algorithm is used both for search of mathematical equation structure and optimal parameters. Each ordered set of variation vectors $\boldsymbol{W}^i$ we generate a bit string $\mathbf{y}^i = \left[y_1^i \ldots y_M^i\right]^T$, $y_j^i \in \{0,1\}$, $j = \overline{1,M}$, $i = \overline{1,H}$, M is the length of bit string $M = P(M_1 + M_2)$, where M_1 is the number of bits for integer part, M_2 is the number of bits for fractional part.

To obtain the values of parameter vector $\mathbf{q}^i = \left[q_1^i \ldots q_P^i\right]^T$ from code $\mathbf{y}^i = \left[y_1^i \ldots y_M^i\right]^T$ the following equations are used:

$$q_k^i = \sum_{j=1}^{M_1+M_2} 2^{M_1-j} z^i_{j+(k-1)(M_1+M_2)}, \qquad (5.1)$$

$k = \overline{1,P}$.

where

$$z_j^i = \begin{cases} y_j^i, \text{ if } (j-1) \bmod (M_1+M_2) = 0 \\ y_j^i \oplus q_{j-1}^i, \text{ otherwise} \end{cases}, \qquad (5.2)$$

$j = \overline{1, P(M_1+M_2)}$.

Each chromosome consists of structural $\boldsymbol{W}^i$ and parametric $\mathbf{y}^i$ parts. To create new chromosomes we randomly choose two chromosomes $\left(\boldsymbol{W}^{j_1}, \mathbf{y}^{j_1}\right)$, $\left(\boldsymbol{W}^{j_2}, \mathbf{y}^{j_2}\right)$ and make a crossover. As a result of crossover we obtain four new chromosomes $\left(\boldsymbol{W}^{H+i}, \mathbf{y}^{H+i}\right)$, $i = \overline{1,4}$. For two chromosomes we cross only parametric parts, and structural parts remain the same. In two other chromosomes we cross structural and parametric parts

$$\mathbf{y}^{H+1} = \left[y_1^{j_1} \dots y_{s-1}^{j_1} y_s^{j_2} \dots y_M^{j_2} \right]^T, \quad (5.3)$$

$$\boldsymbol{W}^{H+1} = \boldsymbol{W}^{j_1}, \quad (5.4)$$

$$\mathbf{y}^{H+2} = \left[y_1^{j_2} \dots y_{s-1}^{j_2} y_s^{j_1} \dots y_M^{j_1} \right]^T, \quad (5.5)$$

$$\boldsymbol{W}^{H+2} = \boldsymbol{W}^{j_2}, \quad (5.6)$$

$$\mathbf{y}^{H+3} = \left[y_1^{j_1} \dots y_{s-1}^{j_1} y_s^{j_2} \dots y_M^{j_2} \right]^T, \quad (5.7)$$

$$\boldsymbol{W}^{H+3} = \left(\mathbf{w}^{1,j_1}, \dots, \mathbf{w}^{d-1,j_1}, \mathbf{w}^{d,j_2}, \dots, \mathbf{w}^{l,j_2} \right), \quad (5.8)$$

$$\mathbf{y}^{H+4} = \left[y_1^{j_2} \dots y_{s-1}^{j_2} y_s^{j_1} \dots y_M^{j_1} \right]^T, \quad (5.9)$$

$$\boldsymbol{W}^{H+4} = \left(\mathbf{w}^{1,j_2}, \dots, \mathbf{w}^{d-1,j_2}, \mathbf{w}^{d,j_1}, \dots, \mathbf{w}^{l,j_1} \right), \quad (5.10)$$

where s and d are two randomly chosen crossover points.

In the problem of multi-objective synthesis (2.1)-(2.8) each chromosome $\left(W^j, \mathbf{y}^j \right)$ corresponds to NOM $\boldsymbol{\Psi}^j$ or solution (2.3) $\mathbf{u}^j = g^j(\mathbf{x}, \mathbf{q})$. To estimate chromosome j we use the value that shows the number of chromosome in the generation that are better than the chromosome j in terms of correlation of Pareto

$$\Lambda_j = \sum_{i=1}^{H} \lambda_j(i), \quad (5.7)$$

where

$$\lambda_j(i) = \begin{cases} 1, \text{ if } \mathbf{J}(\mathbf{u}^i) \le \mathbf{J}(\mathbf{u}^j) \\ 0, \text{ otherwise} \end{cases}. \quad (5.8)$$

Value Λ_j is called the distance from the chromosome j to Pareto set. If $\Lambda_j = 0$ that means that the chromosome belongs to Pareto set and can be considered as one of solutions to (2.1)-(2.6). Final decision concerning the single solution from Pareto set needs additional research.

VI. AN EXAMPLE OF SYNTHESIS

Consider the problem of control for space vehicle descent in atmosphere of the Earth. The mathematical model of control object is described by following differential equations

$$\dot{x}_1 = x_2, \quad (6.1)$$

$$\dot{x}_2 = -\frac{g_0 R_z^2 x_1}{\left(x_1^2 + x_3^2 \right)^{3/2}} - \frac{(1+u) S \rho_0}{m} \cdot x_2 \sqrt{x_2^2 + x_4^2}\, e^{-\lambda\left(\sqrt{x_1^2 + x_3^2} - R_z \right)}, \quad (6.2)$$

$$\dot{x}_3 = x_4, \quad (6.3)$$

$$\dot{x}_4 = -\frac{g_0 R_z^2 x_3}{\left(x_1^2 + x_3^2 \right)^{3/2}} - \frac{(1+u) S \rho_0}{m} \cdot x_4 \sqrt{x_2^2 + x_4^2}\, e^{-\lambda\left(\sqrt{x_1^2 + x_3^2} - R_z \right)}, \quad (6.4)$$

where x_1, x_3 are coordinates of the centre of gravity of a space vehicle in orthogonal geocentric system, x_2, x_4 are components of speed of a space vehicle, u is a control, m is a weight if space vehicle, g_0 is acceleration of free falling on a surface of the Earth, R_z is the radius of the Earth, S is the area of surface of air resistance, ρ_0 is atmosphere density on a surface of the Earth, λ - factor air rarity.

The control has the constrains $u^- \le u \le u^+$. The control changes the area of aerodynamic surface resistance from $(1+u^-)S$ to $(1+u^+)S$.

For the system are set initial conditions (see Fig. 6.1)

$$x_1(0) = 0,\ x_2(0) = V_0 \cos\beta_0,\ x_3(0) = R_z + h_0,\ x_4(0) = V_0 \sin\beta_0, \quad (6.5)$$

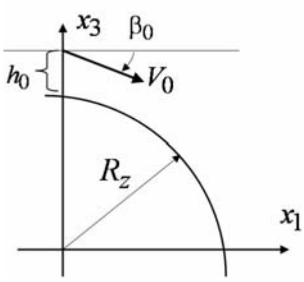

Fig. 6.1. Initial conditions

where V_0 is absolute value of initial speed, h_0 is initial altitude, β_0 is initial angle of slope to a horizon plane

$$\beta_0^- \le \beta_0 \le \beta_0^+ . \qquad (6.6)$$

It is necessary to minimize the following functional

$$J_1 = \Delta\beta_0 \sum_{i=0}^{K} \left\{ \max_t F_g(t) \right\}_{\beta_0 = \beta_0^- + i\Delta\beta_0} , \qquad (6.7)$$

$$J_2 = \Delta\beta_0 \sum_{i=0}^{K} \left\{ R_z \mathrm{arctg} \frac{x_4(t_f)}{x_2(t_f)} - L_f \right\}_{\beta_0 = \beta_0^- + i\Delta\beta_0} ,\backslash \qquad (6.8)$$

where $F_g(t)$ is value of G-force

$$F_g(t) = \frac{\left(x_1^2(t) + x_3^2(t)\right)\sqrt{\dot{x}_2^2(t) + \dot{x}_4^2(t)}}{R_z g_0} , \qquad (6.9)$$

$\Delta\beta_0$ is the step of integration, K is the quantity of intervals for integration, t_f is terminal time, $t_f = t$, if $\left|\sqrt{x_1^2(t) + x_3^2(t)} - (R_z + h_f)\right| = 0$, L_f is the given size of horizontal range, h_f is the given terminal altitude.

Control should depend on problem space coordinates $u = g(\mathbf{x}, \mathbf{q})$.

To solve this problem we used network operator method and genetic algorithm.

To construct the basic solution we draw a line between initial and terminal points

$$\frac{h - h_0}{h_f - h_0} = \frac{L - L_0}{L_f - L_0} , \qquad (6.10)$$

where

$$h = \sqrt{x_1^2 + x_3^2} - R_z . \qquad (6.11)$$

$$L = R_z \mathrm{arctg} \frac{x_1}{x_3} , \qquad (6.12)$$

We transfer location and angle deviations from the trajectory to the control system inputs

$$v_1 = \frac{\sqrt{x_1^2 + x_3^2} - R_z}{h_0} - \left(\left(\frac{h_f / h_0 - 1}{L_f} \right) R_z \mathrm{arctg} \frac{x_1}{x_3} + 1 \right) ,$$

(6.13)

$$v_2 = \mathrm{arctg} \left(\frac{(x_1 x_2 + x_3 x_4)\sqrt{x_1^2 + x_3^2}}{R_z (x_2 x_3 - x_1 x_4)} \right) -$$

$$- \mathrm{arctg} \left(\frac{\sqrt{x_1^2 + x_3^2} - R_z - h_f}{L_f - R_z \mathrm{arctg} \frac{x_1}{x_3}} \right) , \qquad (6.14)$$

To construct the network operator we choused the following constructive sets:

$\mathbf{X} = (v_1, v_2)$, $\boldsymbol{Q} = (q_1, \ldots, q_4)$, $\boldsymbol{O}_1 = (\rho_1(z), \ldots, \rho_8(z))$,

$\boldsymbol{O}_2 = (\chi_0(z'', z'') = z' + z'', \chi_1(z'', z'') = z'z'')$,

where $\rho_1(z) = z$, $\rho_2(z) = -z$, $\rho_3(z) = \mathrm{sgn}(z)\sqrt{|z|}$, $\rho_4(z) = \frac{1}{z}$, $\rho_5(z) = \frac{1 - e^{-z}}{1 + e^{-z}}$, $\rho_6(z) = \theta(z) = \begin{cases} 1, \text{ if } z \ge 0 \\ 0, \text{ other} \end{cases}$, $\rho_7(z) = z^3$, $\rho_8(z) = \sqrt[3]{z}$.

We write the basic solution in the following form

$$u = \begin{cases} u^+, \text{ if } y > u^+ \\ u^-, \text{ if } y < u^- \\ y, \text{ other} \end{cases} , \qquad (6.15)$$

where

$$y = q_3(q_2 v_2 - q_1 v_1) - q_2 v_2 - q_1 v_1 + q_4 , \qquad (6.16)$$

$q_1 = 1$, $q_2 = 1$, $q_3 = 1$, $q_4 = 1$.

The basis control at initial condition $\beta_0 = -0.09$ gives terminal deviation $|L - L_f| = 92545$ m and maximum G-Force $F_g = 8.15$.

Parameters of genetic algorithm were the following: population size $H = 512$, number of generations $G = 127$, number of crossed pairs in one generation $R = 256$, number of generations between epochs or change of the basis solution $E = 10$, length of a structural part of a chromosome $l = 8$, number of adjustable parameters $P = 4$, number of bits for integer part $c = 2$, number of bits for fractional part $d = 6$, dimension of the network operator matrix $L = 16$, probability of mutation $p_m = 0{,}8$.

At calculations the following parameters of model were used: $h_0 = 100000$ m; $h_f = 10000$ m; $L_f = 14500000$ m; $R_z = 6371000$ m; $\rho_0 = 1{,}22$ kg/m^3; $m = 5000$ kg; $S = 3{,}5$ m^2; $g_0 = 9{,}81$ m/c^2; $\lambda = 1{,}35 \cdot 10^{-4}$ m^{-1}; $V_0 = 11200$ m/c; $\beta_0^- = -0{,}09$; $\beta_0^+ = -0{,}085$; $\Delta\beta_0 = 0{,}001$; $u^- = -0{,}2$; $u^+ = 0{,}2$.

The obtained Pareto set is presented on fig 6.2

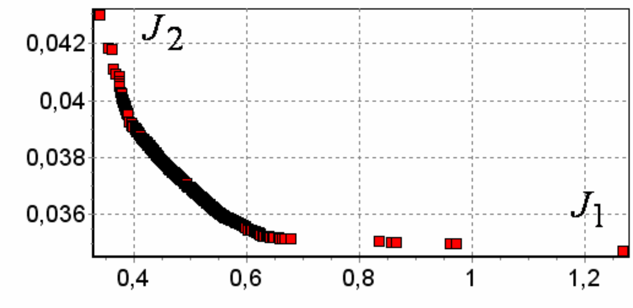

Fig. 6.2. The set of Pareto

We select one solution on the Pareto set. Matrix of the network operator for the selected solution has the following form

$$\mathbf{\Psi} = \begin{bmatrix}
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&5&0&0&0\\
0&0&0&0&0&0&2&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&3&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&6\\
0&0&0&0&0&0&0&0&1&0&1&0&0&1&0&6\\
0&0&0&0&0&0&0&1&1&0&2&0&0&2&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&1&8&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{bmatrix}.$$

This solution corresponds to the control

$$y = \sqrt[3]{\frac{1-e^{-v_2}}{1+e^{-v_2}} z_{10} z_{12}} + \frac{1-e^{-v_2}}{1+e^{-v_2}} z_{10} z_{12} + $$
$$+ v_1 - q_1 - q_2 v_2 + 1 + \theta(v_1 - q_1),$$

where $z_{10} = v_1 - q_1 + q_2 v_2 + \sqrt{q_3}$, $z_{12} = q_4 + v_1 - q_1 - q_2 v_2$, $q_1 = 3.03125$, $q_2 = 3.671875$, $q_2 = 0.296875$, $q_4 = 2.234375$.

The results of simulation for $\beta_0 = -0.09$ are represented on figs 6.3, 6.4.

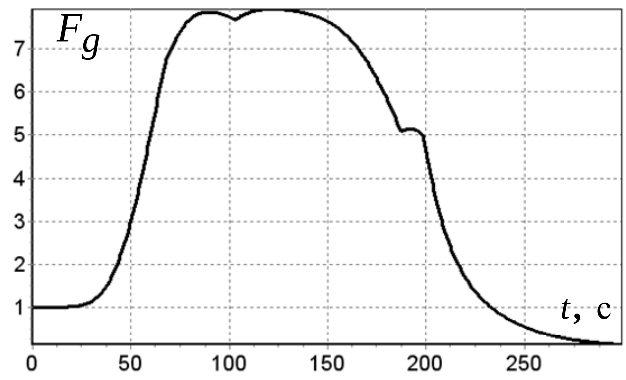

Fig 6.3. G-Force

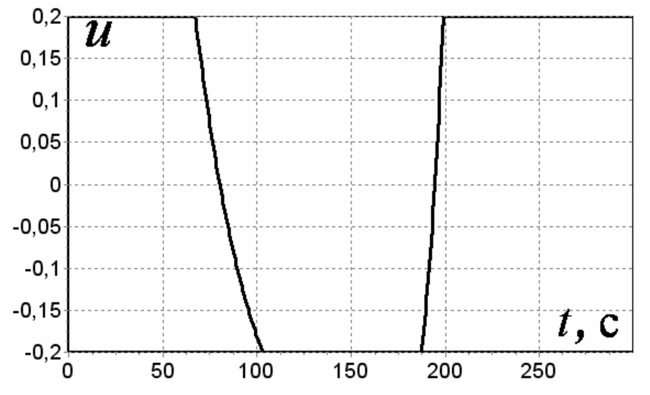

Fig. 6.4. The control u

The received control at initial condition $\beta_0 = -0.09$ gives a value of terminal deviation $|L - L_f| = 287.7\,\text{m}$ and maximum G-Force $F_g = 7.93$.

The other solution has the following NOM

$$\Psi = \begin{bmatrix}
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&3&0&0&0&5&0&0&0\\
0&0&0&0&0&0&2&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&3&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&6\\
0&0&0&0&0&0&0&0&1&0&1&6&0&1&0&6\\
0&0&0&0&0&0&0&1&1&0&2&0&0&2&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&8&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{bmatrix}$$

and the following control

$$y = \sqrt[3]{\frac{1-e^{-v_2}}{1+e^{-v_2}}}\, z_{10} z_{12} + v_1 - q_1 - q_2 v_2 + 1 + \theta(v_1 - q_1),$$

where $z_{10} = v_1 - q_1 + q_2 v_2 + \operatorname{sgn}(v_2)\sqrt{|v_2|} + \sqrt{q_3}$,

$z_{12} = q_4 + \theta(v_1 - q_1) + v_1 - q_1 - q_2 v_2$,

$q_1 = 3.015625$, $q_2 = 3.984375$, $q_2 = 0.796875$, $q_4 = 2.21875$.

This control at initial condition $\beta_0 = -0.09$ gives a value of terminal deviation $|L - L_f| = 1258.1\,\text{m}$ and maximum G-Force $F_g = 7.937$.

VII. CONCLUSIONS

The network operator allows to synthesize the law of control like function of problem space coordinates. The network operator describes mathematical expression in form of an integer matrix. With the help of the network operator the method for solution of the problem of multi-objective synthesis of optimal control was developed.

In example the given law of control for a space unit is nonlinear and complex. The control provides satisfactory terminal accuracy and G-Force.

VIII. AKNOWLEDGMENT

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