The Synthesis of Optimal Control System by the Network Operator Method

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Abstract: The problem of synthesis of optimal control system is considered. It is proved that generally it is necessary to search for control as function of not only coordinates of space of conditions, but of velocities of changes of these coordinates. To solve the problem of synthesis it is offered to use the network operator that allows to present mathematical expression of functional dependence in the form of the directed graph. Properties of the network operator are given. For the search of optimal functional dependence on the set of network operators the genetic algorithm is used. The practical example of the solution of a problem of synthesis is given.

Keywords: Optimal control synthesis, nonlinear control systems, network operator, genetic programming, principle of basic solution

1. INTRODUCTION

It is considered that the problem of synthesis of a control system consists in finding control in the form of functional dependence of coordinates of space of conditions. Known methods of the solution of the problem of synthesis are based on the solution of Bellman equation at the expense of use of properties of model of system and functional. If we use Bellman equations the mathematical model of system should satisfy additional conditions in the form of smoothness of the right parts of the differential equations or unlimited control. As a result we receive control in the form of function of coordinates of space of conditions.

However in general control as a functional dependence of coordinates of space of conditions is not enough for the solution of a problem of synthesis. Method of genetic programming (Koza J.R., 1992) for solution of the synthesis problem (Keane M.A. *et al.*, 2002) is known.

In article it is shown that for the solution of a problem of synthesis it is necessary to search the control that depends not only on coordinates of spaces of conditions, but also on velocity of change of these coordinates.

For the problem solution we use a method of network operator (Diveyev A.I., Sofronova E.A., 2008) which allows to represent mathematical expressions in the form of the directed graph and to describe it by means of an integer matrix. For calculation of derivatives of coordinates of space of conditions we use numerical differentiation.

2. PROBLEM STATEMENT

We will consider a problem of synthesis of systems of automatic control. Let the optimal control problem is formulated. The mathematical model of object of control is set

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathrm{u}), \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{u} \in \boldsymbol{U} \subseteq \mathbb{R}^m$, $\boldsymbol{U}$ - a limited set,
$\mathbf{f}(\mathbf{x},\mathbf{u})$: $\mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$, $m \le n$.

For system (1) initial values are set

$$\mathbf{x}(0) = \mathbf{x}^0 = \left[x_1^0 \dots x_n^0\right]^T. \tag{2}$$

Terminal conditions are set

$$\mathbf{x}(t_f) = \mathbf{x}^f = \left[x_1^f \dots x_n^f\right]^T, \tag{3}$$

where t_f - given time of control.

Functional of quality is

$$J = F\left(\mathbf{x}(t_f)\right) + \int_0^{t_f} f_0(\mathbf{x}(t), \mathbf{u}(t))dt. \tag{4}$$

It is necessary to find the admissible control that satisfies the restrictions

$$\mathbf{u} \in \boldsymbol{U}, \tag{5}$$

$$\mathbf{u}(\cdot) \in KC[0, t_f], \tag{6}$$

where $KC[0, t_f]$ - class of piecewise continuous functions, set on an interval $[0, t_f]$.

If we solve the problem of synthesis then the control is a function of coordinates of space of conditions

$$\mathbf{u} = \mathbf{g}(\mathbf{x}, \dot{\mathbf{x}}), \tag{7}$$

where $\mathbf{g}(\mathbf{x}, \dot{\mathbf{x}})$: $\mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^m$.

3. PROPERTY OF THE SOLUTION

Let us show that if there is a solution of the problem of optimal control there is a solution of a problem of synthesis.

Theorem 1. For the solution of the problem (1)-(5), (7) $\mathbf{u} = \mathbf{g}(\mathbf{x}, \dot{\mathbf{x}})$, where $\mathbf{g}(\mathbf{x}, \dot{\mathbf{x}})$: $\mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^m$, such that $\mathbf{u} = \mathbf{g}(\tilde{\mathbf{x}}(t), \dot{\tilde{\mathbf{x}}}(t)) \in \boldsymbol{U}$, $0 \le t \le t_f$, $\mathbf{x}(t_f) = \mathbf{x}^f$ and $J(\mathbf{g}(\tilde{\mathbf{x}}(\cdot), \dot{\tilde{\mathbf{x}}}(\cdot))) = J(\tilde{\mathbf{u}}(\cdot))$, $\tilde{\mathbf{x}}(\cdot) = (\tilde{\mathbf{x}}(t), 0 \le t \le t_f)$, the performance of following conditions is necessary and enough:

a) there is a function $\tilde{\mathbf{u}}(t): \mathbb{R}^m \to \mathbb{R}^1$, $\tilde{\mathbf{u}}(\cdot) \in KC[0, t_f]$, such that $\tilde{\mathbf{u}}(t) \in \boldsymbol{U}$, $0 \le t \le t_f$, and

$$J(\tilde{\mathbf{u}}(\cdot)) = \min_{\mathbf{u} \in \boldsymbol{U}} \left\{ F(\tilde{\mathbf{x}}(t_f)) + \int_0^{t_f} f_0(\tilde{\mathbf{x}}(t), \tilde{\mathbf{u}}(t)) dt \right\},$$

where $\tilde{\mathbf{x}}(t)$ - value of the solution of system (1) at the moment t with function $\tilde{\mathbf{u}}(\cdot)$ and initial conditions (2).

b) $\forall t \in [0, t_f]$, $\exists \boldsymbol{\delta}(t) \in \mathbb{R}^m$, $\|\boldsymbol{\delta}(t)\| \ge M > 0$, $\Rightarrow \mathbf{u}(t) = \tilde{\mathbf{u}}(t) + \boldsymbol{\delta}(t)$, $\operatorname{rank} \dfrac{\partial \mathbf{f}(\tilde{\mathbf{x}}(t), \mathbf{u}(t))}{\partial \mathbf{u}} = m$, $0 \le t \le t_f$.

The proof. Let us substitute function $\tilde{\mathbf{u}}(t)$ in system of the differential equations (1) and solve it at initial values (2), we will get $\tilde{\mathbf{x}}(\cdot)$. From the condition b follows that system of the equations $\mathbf{f}(\tilde{\mathbf{x}}(t), \mathbf{u}) - \dot{\tilde{\mathbf{x}}}(t) = 0$ has solution for $\mathbf{u}$. It corresponds to the function $\mathbf{g}(\mathbf{x}(t), \dot{\mathbf{x}}(t))$ with parameter t. From the condition a follows that $\tilde{\mathbf{u}}(t) \in \boldsymbol{U}$, $0 \le t \le t_f$, therefore the condition is satisfied $\mathbf{g}(\tilde{\mathbf{x}}(t), \dot{\tilde{\mathbf{x}}}(t)) \in \boldsymbol{U}$, and as from the condition a of theorem, follows $J(\tilde{\mathbf{u}}(\cdot)) = \min_{\mathbf{u} \in \boldsymbol{U}} J$, that we obtain that $J(\mathbf{g}(\tilde{\mathbf{x}}, \dot{\tilde{\mathbf{x}}})) = \min_{\mathbf{u} \in \boldsymbol{U}} J$.

Let us show that control as a function of coordinates of space of conditions $\mathbf{u} = \mathbf{h}(\mathbf{x})$ is not enough for the solution of a problem of synthesis (1) - (5), (7).

Suppose that for function $\tilde{\mathbf{x}}(t)$ the following conditions are satisfied: $\exists t', t'' \in [0, t_f]$, $t' \ne t''$, $\tilde{\mathbf{x}}(t') = \tilde{\mathbf{x}}(t'')$, $\dot{\tilde{\mathbf{x}}}(t') \ne \dot{\tilde{\mathbf{x}}}(t'')$. Then for any function $\mathbf{u} = \mathbf{h}(\mathbf{x})$, $\mathbf{h}(\mathbf{x}): \mathbb{R}^n \to \mathbb{R}^m$, we receive that $\mathbf{u}(t') = \mathbf{h}(\tilde{\mathbf{x}}(t'))$, $\mathbf{h}(t'') = \mathbf{h}(\tilde{\mathbf{x}}(t'))$, therefore $\mathbf{u}(t'') = \mathbf{u}(t')$.

At the same time for initial system (1) we receive inequalities $\dot{\tilde{\mathbf{x}}}(t'') \ne \mathbf{f}(\tilde{\mathbf{x}}(t'), \mathbf{u}(t'))$, $\dot{\tilde{\mathbf{x}}}(t') \ne \mathbf{f}(\tilde{\mathbf{x}}(t''), \mathbf{u}(t''))$, therefore one of values $\mathbf{u}$ for points t' or t'' is not optimal, i.e. conditions $\mathbf{h}(\tilde{\mathbf{x}}(t')) \ne \tilde{\mathbf{u}}(t')$ or $\mathbf{h}(\tilde{\mathbf{x}}(t'')) \ne \tilde{\mathbf{u}}(t'')$ are satisfied.

Since $\mathbf{u} = \mathbf{h}(\tilde{\mathbf{x}})$ is not optimal then terminal conditions are not satisfied $\mathbf{x}(t_f) \ne \mathbf{x}^f$ or value of functional is not minimum $J(\tilde{\mathbf{u}}(\cdot)) < J(\mathbf{h}(\tilde{\mathbf{x}}))$, i.e. the problem of synthesis (1) - (5), (7) has no solution with control $\mathbf{u} = \mathbf{h}(\mathbf{x})$. ■

4. THE NETWORK OPERATOR

To construct an algorithm of the solution of a problem of synthesis (1) - (5), (7) we use a method of the network operator (Diveyev A.I., Sofronova E.A., 2008). Let us take into consideration some finite ordered sets that consist of the elements that compose mathematical expression.

The set of variables is the ordered set of elements of a vector of conditions of the object,

$$\boldsymbol{V} = (v_1, \ldots, v_P),\ v_i \in \mathbb{R}^1,\ i = \overline{1, P}. \tag{8}$$

The set of parameters is the ordered set of elements of a vector of parameters,

$$\boldsymbol{C} = (c_1, \ldots, c_R),\ c_i \in \mathbb{R}^1,\ i = \overline{1, R}. \tag{9}$$

The set of unary operations is the ordered set of functions or the single-valued transformation set on numerical set,

$$\boldsymbol{O}_1 = (\rho_1(z), \rho_2(z), \ldots, \rho_W(z)). \tag{10}$$

where $\rho_i(z)$: $\mathbb{R}^1 \to \mathbb{R}^1$, $\mathbb{R}^1$, $\exists y \in \mathbb{R}^1 \Rightarrow y = \rho_i(z)$, $i = \overline{1,W}$.

The set of binary operations is the ordered set of functions of two arguments or single-valued transformation Cartesian product of pair identical numerical sets in one same numerical set,

$$\boldsymbol{O}_2 = (\chi_0(z',z''), \chi_1(z',z''), \ldots, \chi_{V-1}(z',z'')) \quad (11)$$

where $\chi_i(z',z'')$: $\mathbb{R}^1 \times \mathbb{R}^1 = \mathbb{R}^2 \to \mathbb{R}^1$, $\forall z', z'' \in \mathbb{R}^1$, $\exists y \in \mathbb{R}^1 \Rightarrow y = \chi_i(z',z'')$, $i = \overline{0,V-1}$.

All binary operations have following properties:

- commutativity

$$\chi_i(z',z'') = \chi_i(z'',z'),\ \chi_i \in \boldsymbol{O}_2,\ i = \overline{1,V}; \quad (12)$$

- associativity

$$\chi_i(\chi_i(z',z''),z''') = \chi_i(z',\chi_i(z'',z''')),$$

$$\chi_i \in \boldsymbol{O}_2,\ i = \overline{1,V}; \quad (13)$$

- have unity element

$$\forall e_i \in \mathbb{R}^1 \quad \chi_i(e_i,z) = \chi_i(z,e_i) = z, \quad \chi_i \in \boldsymbol{O}_2, \quad i = \overline{1,V}. \quad (14)$$

Network operator is the directed graph that has the following properties:

a) graph has no loops;

b)any nonsource node has at least one edge from the source node;

c) any non sink node has at least one edge to sink node;

d) every source node corresponds to the element of set of variables $\boldsymbol{V}$ or of set of parameters $\boldsymbol{C}$;

e) every node corresponds to binary operation of set of binary operations $\boldsymbol{O}_2$;

f) every edge corresponds to unary operation of set of unary operations $\boldsymbol{O}_1$.

Any network operator corresponds to some mathematical expression that can be written with the help of variables, constants, unary and binary operations used in the network operator.

To present the network operator in memory of the computer we use an integer network operator matrix (NOM). NOM is an integer matrix where binary operations are located in diagonal elements, and other elements are zeros or numbers of unary operations. If we replace diagonal elements by zeros, and nonzero not diagonal elements on ones we receive an incident matrix of the graph of the network that satisfies conditions a-c in definition of the network operator.

Let us number the nodes of the network operator so that the number of the node with an outcoming edge would be less than the number of the node which this edge comes in. It can always be done, because according to definition the network operator has no loops. In this case we receive an upper-triangular NOM $\boldsymbol{\Psi} = [\psi_{ij}]$, $\psi_{ij} = 0$, if $i > j$, $i, j = \overline{1,L}$, where L -number of nodes in the network operator.

To link the parameters or variables to certain source nodes and result values to sink nodes, we should define additional integer vectors:

- vector of numbers of nodes for input variables $\mathbf{b} = [b_1 \ldots b_P]^T$, where b_i - number of source node in the network operator with which the variable $v_i, i = \overline{1,P}$ is connected;

- vector of numbers of nodes for parameters $\mathbf{s} = [s_1 \ldots s_R]^T$, where s_i - number of source node in the network operator with which the parameter c_i, $i = \overline{1,R}$ is connected;

- vector of numbers of nodes for output variables $\mathbf{d} = [d_1 \ldots d_m]^T$, where d_i - number of sink node in the network operator with which the output variable y_i, $i = \overline{1,m}$ is connected. The condition $m > 1$ means that the network operator describes more than one function.

Let us introduce the node vector $\mathbf{z} = [z_1 \ldots z_L]^T$ of size L which is equal to the number of nodes in the network operator.

For calculation of mathematical expression by means of NOM $\boldsymbol{\Psi}$ we set initial values of the node vector

$$z_i^{(0)} = \begin{cases} v_k, \text{ if } i = b_k,\ k = \overline{1,n} \\ c_j, \text{ if } i = s_j,\ j = \overline{1,l} \\ e_{\psi_{ii}}, \text{ otherwise} \end{cases}, \quad (15)$$

$i = \overline{1,L}$.

where e_k - unit element for binary operation χ_k.

Look through the elements of NOM $\boldsymbol{\Psi}$, that are over the main diagonal. If $\psi_{ij} \neq 0$, then

$$z_j^{(i)} = \chi_{\Psi_{jj}}\left(z_j^{(i-1)}, \rho_{\Psi_{ij}}\left(z_i^{(i-1)}\right)\right), \qquad i = \overline{1, L-1},$$
$$j = \overline{i+1, L}.$$

Elements $z_i^{(L-1)}$, $i = d_k$, $k = \overline{1,m}$, contain the values of calculation of mathematical expressions that network operator describes.

Consider an example. Suppose the following mathematical expressions are given

$$y_1 = ax_1x_2 + bx_2^2,\ y_2 = ax_1^3 + bx_2^3.$$

Define the set of variables $\boldsymbol{V} = (x_1, x_2)$, the set of parameters $\boldsymbol{C} = (a, b)$, the set of unary operations $\boldsymbol{O}_1 = \left(\rho_1(z) = z, \rho_2(z) = z^2, \rho_3(z) = z^3\right)$ and the set of binary operations $\boldsymbol{O}_2 = \left(\chi_0(z', z'') = z' + z'', \chi_1(z', z'') = z'z''\right)$.

Network operator for given mathematical expressions is shown on Fig. 1

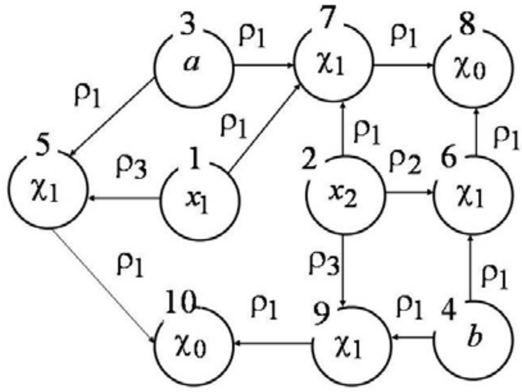

Fig. 1. Example of network operator

Network operator on Fig.1 contains input variables or parameters in source nodes or binary operations. On edges of the graph unary operations are located. Number the nodes so that the number of the node with an outcoming edge would be less than the number of the node which this edge comes in. Such numeration is shown on Fig. 1 in the top of the nodes. Suppose that the numbers of nodes are numbers of rows in matrix, construct the NOM, we get the following

$$\boldsymbol{\Psi} = \begin{bmatrix} 0&0&0&0&3&0&1&0&0&0 \\ 0&0&0&0&0&2&1&0&0&0 \\ 0&0&0&0&1&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0&1&0 \\ 0&0&0&0&1&0&0&0&0&1 \\ 0&0&0&0&0&1&0&1&0&0 \\ 0&0&0&0&0&0&1&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&1 \\ 0&0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

Vectors of numbers of nodes for variables, parameters and outputs for given example are $\mathbf{b} = [1\ 2]^T$. $\mathbf{s} = [3\ 4]^T$, $\mathbf{d} = [8\ 10]^T$.

Initial values of node vector according to (15) are $\mathbf{z}^{(0)} = [x_1\ x_2\ a\ b\ 1\ 1\ 1\ 0\ 1\ 0]^T$, where 1 - unit element for multiplication $\chi_1(z', z'')$, 0 - unit element for addition $\chi_0(z', z'')$.

Results of calculation are obtained after going though all rows in NOM $\boldsymbol{\Psi}$. They are situated in elements of vector of nodes that correspond to outputs. If we omit the operation of identity $\rho_1(z)$ mathematical equations will be the following:

$$z_8^{(10)} = \chi_0(\chi_1(a, \chi_1(x_1, x_2)), \chi_1(b, \rho_2(x_2))),$$
$$z_{10}^{(10)} = \chi_0(\chi_1(a, \rho_3(x_1)), \chi_1(b, \rho_3(x_2))).$$

5. GENETIC ALGORITHM AND BASIC SOLUTION

To construct the algorithm of optimal control synthesis we use a principle of basic solution. The principle of basic solution means that when solving optimization problems, initially we set the basic solution that is one of admissible solutions, then define small variations of basic solution and create search algorithm that searches for the optimal solution on the set of small variations.

Let us define the following small variations of network operator: 0 - replacement of unary operation by the edge of the graph; 1 - replacement of binary operation in the node; 2 - addition of the edge with unary operation; 3 - removal of unary operation with the edge of the graph.

To describe any variation it is enough to use an integer vector of four elements $\mathbf{w} = [w_1\ w_2\ w_3\ w_4]^T$. The first element of vector of variations $\mathbf{w}$ specifies the number of variation, the second – the number of node that the edge comes out, the third - the number of node that the edge comes in, and the fourth – number of unary or binary operation.

For example $\mathbf{w} = [0\ \ 2\ \ 7\ \ 3]^T$ then

$$\mathbf{w}\circ\mathbf{\Psi}=\begin{bmatrix}0&0&0&0&3&0&1&0&0&0\\0&0&0&0&0&2&3&0&0&0\\0&0&0&0&1&0&1&0&0&0\\0&0&0&0&0&1&0&0&1&0\\0&0&0&0&1&0&0&0&0&1\\0&0&0&0&0&1&0&1&0&0\\0&0&0&0&0&0&1&1&0&0\\0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&1&1\\0&0&0&0&0&0&0&0&0&0\end{bmatrix}.$$

This NOM corresponds to the following expression

$$z_8^{(10)}=\chi_0(\chi_1(a,\chi_1(x_1,\rho_3(x_2))),\chi_1(b,\rho_2(x_2)))$$

or

$$y_1=ax_1x_2^3+bx_2^2.$$

For genetic algorithm we should create a basic solution that is described by NOM $\mathbf{\Psi}^0=\left[\psi_{ij}^0\right]$, $i,j=\overline{1,L}$. The ordered set of vectors of variations is considered to be a chromosome

$$\boldsymbol{W}^i=\left(\mathbf{w}^{i1},\ldots,\mathbf{w}^{il}\right),\ i=\overline{1,H},\tag{16}$$

where H is the size of population, l is the length of chromosome.

All genetic operations are performed on the vectors of variations. Each vector of variations $\boldsymbol{W}^i$ changes basic solution thus we obtain new solution

$$\mathbf{\Psi}^i=\mathbf{w}^{i1}\circ\ldots\circ\mathbf{w}^{il}\circ\mathbf{\Psi}^0\tag{17}$$

For effective search we change basic solution on the best solution found after some number of generations called epoch.

For search of mathematical expression which structure is described by NOM and optimal values of parameters we use genetic algorithm. For this purpose we generate an ordered set of variation vectors $\boldsymbol{W}^i$ and a bit string $\mathbf{y}^i=\left[y_1^i\ldots y_M^i\right]^T$, $y_j^i\in\{0,1\}$, $j=\overline{1,M}$, $i=\overline{1,H}$, where M is the length of the bit string.

$$M=(M_1+M_2)R,\tag{18}$$

where M_1 is number of bit for integer part, M_2 is number of bit for fractional part.

For obtain the values of a vector of parameters $\mathbf{c}^i=\left[c_1^i\ldots c_R^i\right]^T$ by Grey code $\mathbf{y}^i=\left[y_1^i\ldots y_M^i\right]^T$ we carry out calculations by means of following expressions:

$$c_k^i=\sum_{j=1}^{M_1+M_2}2^{M_1-j}q_{j+(k-1)(M_1+M_2)}^i,\tag{19}$$

$k=\overline{1,R}$,

where

$$q_j^i=\begin{cases}y_j^i,\ \text{if}\ (j-1)\bmod(M_1+M_2)=0\\ y_j^i\oplus q_{j-1}^i,\ \text{else}\end{cases},\tag{20}$$

$j=\overline{1,R(M_1+M_2)}$.

Thus in genetic algorithm each chromosome consists of two parts, a structural part $\boldsymbol{W}^i$ and parametrical one $\mathbf{y}^i$.

When crossing two chromosomes we receive four children. Two children are crossed only in parametrical parts, and other two other children are crossed both in structural and parametrical parts.

6. AN EXAMPLE

As an example we consider a control system of the Lotka-Volterra equations

$$\dot{x}_1=\alpha x_1-\beta x_1x_2+u_1,$$

$$\dot{x}_2=\gamma x_1x_2-\delta x_2+u_2.$$

The control has restrictions

$$u_i^-\le u_i\le u_i^+,\ i=1,2$$

For search the control we use the functional

$$J=\int_0^{t_f}\left(|u_1|+|u_2|+\left|x_1-x_1^f\right|+\left|x_2-x_2^f\right|\right)dt$$

We used the following parameters of genetic algorithm: number of chromosomes in population – 100, number of generations – 50, number of couples – 200, number of generations in one epoch – 10, length of chromosome – 8, mutation probability – 0.7, dimension of NOM - 16×16.

We used four parameters, $R=4$. For each parameter integer part was $M_1=0$, and fractional part was $M_2=8$.

For the model we took the following values of constants: $\alpha=1$, $\beta=0.2$, $\gamma=0.5$ $\delta=1$.

The initial conditions were $x_1=1$, $x_2=1$.

The terminal conditions were $x_1^f=2$, $x_2^f=2$. The time of simulation was $t_f=20$.The restrictions for the control were $u_i^-=-1$, $u_i^+=+1$, $i=1,2$.

As a result of synthesis we received the following control

$$u_i = \begin{cases} u_i^+, \text{if } g_i > u_i^+ \\ u_i^-, \text{if } g_i < u_i^- \\ g_i, \text{otherwise} \end{cases}, \ i = 1,2$$

where

$$g_1 = c_3 x_1^3 \operatorname{sign}(\dot{x}_1)\sqrt{\dot{x}_1} + \cos(x_1 + c_1) + \dot{x}_2,$$
$$g_2 = (c_1 + x_1)c_2 x_2 \sin(c_4)\dot{x}_2,$$

$c_1 = 0.3046875$, $c_2 = 0.546875$,

$c_3 = 0.69921875$, $c_1 = 0.08203125$

On Fig 2 the phase portrait of system without control is presented. On Fig 3 the phase portrait of system with obtained control is presented. On Fig. 4 and 5 the components of received control are presented.

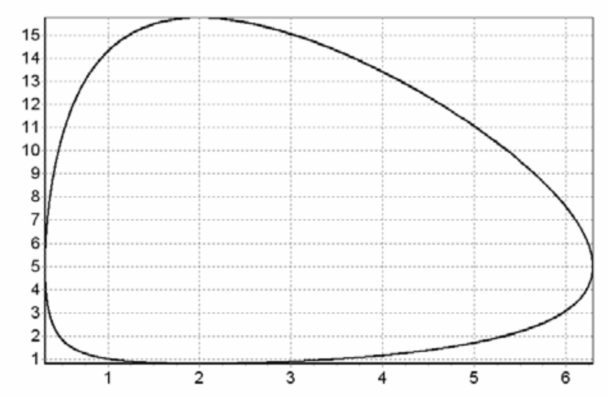

Fig. 2. The phase portrait of the system without control

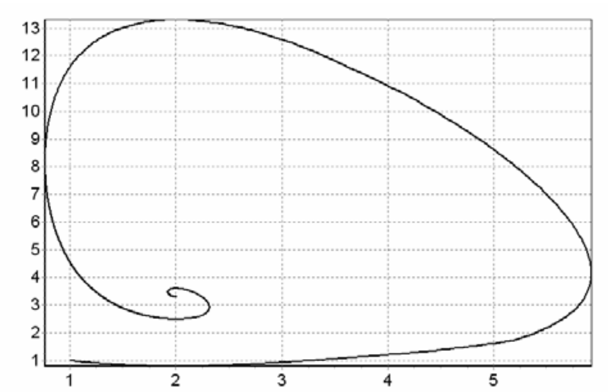

Fig. 3. The phase portrait of the system with obtained control

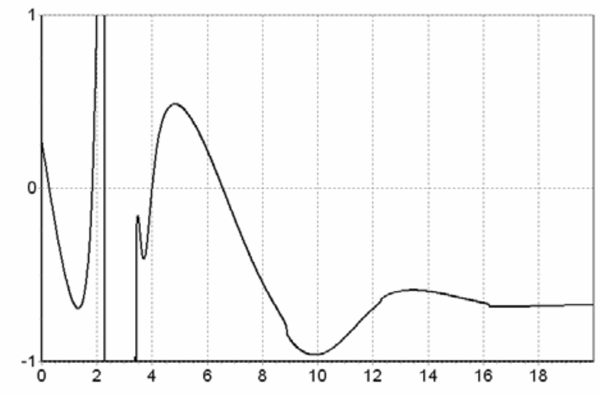

Fig. 4. The control u_1

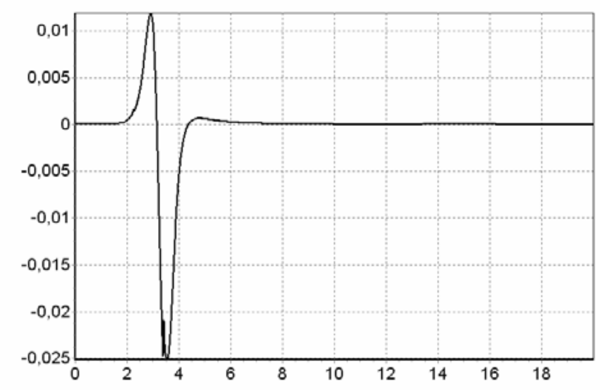

Fig. 5. The control u_2

7. AKNOWLEGMENT

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8. CONCLUSIONS

It is proved that the synthesis of control system is possible, if there is a solution of a problem of optimal control. Application of network operator allows to construct an effective algorithm for synthesis of a control system.

REFERENCES

Koza J.R. (1992) *Genetic Programming: On the Programming of Computers by Means of Natural Selection*. Cambridge, Massachusetts, London, MA:

Koza J.R.,Kean M.A., Streeter M.J., (2002) *Automatic synthesis using genetic programming of improved PID turning rules*, IFAC.

Diveyev A.I., Sofronova E.A. (2008) *Application of network operator method for synthesis of optimal structure and parameters of automatic control system The .proceedings of 17-th IFAC World Congress, Seoul, 2008, 05.07.2008 – 12.07.2008. P. 6106 – 6113.*